Lecture 3

January 12, 2024 11:28 AM

What is the largest number that can be represented using 50 characters?

Call it N

"The largest number that can be represented with 50 characters plus one"

In this course, we are interested in "decision problems" that have binary output where the input is a string.

* Is the given word a palindrome?
* is the given number even?

Notations

* lower case letters a, b, c, d, ... used for symbols/letters
* u, v, w, x, y, z for strings
* α, β, γ ... for patterns
* capital letters A, B, C, D... for sets

Alphabet, Σ: a finite set of symbols

String: finite sequence of symbols

$\Sigma = \{0,1\}$ $\Sigma = \{0,1,2,\dots,9\}$

$\Sigma = \{a,b,c\}$

$x = abc, 123, aab, 011$

$x = \varepsilon$ → null string with length 0

length of a string: $|x|$

$|aba| = 3$
$|\varepsilon| = 0$

concatenation of two strings:

$x = abc$, $y = ab$, $xy = abcab$

a new string

$|xy| = |x| + |y|$

$\varepsilon x = x\varepsilon = x$ "by definition"

$\varepsilon x = x\varepsilon = x$

"by definition"

* Power of a string: $x^0 \triangleq \varepsilon$

$x^{n+1} \triangleq x^n x$

* $\#a(x)$: the total number of occurance of a in x

$\#b(abcbbbb) = 5$

* prefix: we say a string x is a prefix of string y if $\exists z$ such that $y = xz$

$x = abbca \qquad y = abbcada$

x is a prefix of y with $z = da$

* ε is a prefix of any string.
* any string is a prefix of itself

* proper prefix: a prefix that is not the string itself

* ab is a prefix of ab but not a proper prefix.

Sets of Strings

- Σ^* is the set of all strings that can be formed by the alphabet Σ

  - e.g. $\Sigma = \{a, b\}$

    $\Sigma^* = \{\varepsilon, a, b, aa, ab, ba, bb, \ldots\}$

- ϕ: empty set $\phi = \{\}$
- $A = \{ab, a, bba\} \subseteq \Sigma^*$
- $A = \{x \mid x \in \Sigma^* \wedge \#a(x) = 1\}$, $\Sigma = \{a, b\}$

  $= \{a, ab, ba, abb, \ldots\}$

  A is an infinite set, but every string in it has a finite length (by definition)

- Usual operations on sets:

  $A \cap B$, $A \cup B$, $\sim A = \Sigma^* - A$ or $\sim A = \Sigma^* \setminus A$

- Concatenation of sets: A B a set of strings

  $AB \triangleq \{xy : x \in A, y \in B\}$

  $A = \{\varepsilon, aa, b\}$, $B = \{a, b\}$

$AB \triangleq \{xy : x \in A, y \in B\}$

$A = \{a, aa, b\}$, $B = \{a, b\}$

$AB = \{aa, ab, aaa, aab, ba, bb\}$

- $A = \{a, aa\}$ $B = \{\varepsilon, a\}$

$AB = \{a, aa, aa, aaa\} = \{a, aa, aaa\}$

- Powers of sets of strings:

$A^0 \triangleq \{\varepsilon\}$ -> can't be ϕ bc $A\phi = \phi = \phi A$

$A^{n+1} \triangleq A^n A$

- $A^* \triangleq A^0 \cup A^1 \cup A^2 \cup \dots = \bigcup_{i=0}^{\infty} A^i$

↓ Σ^*

$A = \{a, ab\}$

$A^* = \{\varepsilon, a, ab, aa, aab, aba, abab, \dots\}$

- $\phi^* \triangleq \{\varepsilon\}$ $\phi^* A = A$

↳ this definition makes notation cleaner

* We say a binary operation $\otimes$ is associative if

$\forall a, b, c$, we have

$(A \otimes B) \otimes C = A \otimes (B \otimes C)$

commutative if $\forall A, B$,

$A \otimes B = B \otimes A$

Operation	Associative	Commutative	identity
$\cup$	✓	✓	ϕ
$\cap$	✓	✓	U -> or Σ^*
AB	✓	×	$\{\varepsilon\} = \phi^*$